3. The Fractal View: Nature in Mandelbrot's Geometry[1]

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I coined fractal from the Latin fractus, which describes the appearance of a broken stone: irregular and fragmented.[2]
—Benoît Mandelbrot

Nature is rough.[3]
—IBM

Fractal Nature

Is nature "rough"? Since antiquity, philosophers, theologians, and scientists have tried to demonstrate the regularity of nature. Mandelbrot's fractal geometry, by contrast, is based on the proposition that many natural phenomena are so irregular and fragmented that they cannot be described by means of Euclidean geometry.[4] Instead of approaching the forms of nature in the customary way—as regular, three-dimensional bodies—Mandelbrot made use of strange constructions that can no longer be accommodated within our familiar spatial dimensions: endlessly long curves that are nowhere smooth and in magnification constantly reveal new twists.[5] It was Mandelbrot's pathbreaking idea to call these mathematical oddities the fundamental model of nature.[6] A number of physicists, biologists, and geographers recognized the subversiveness in his proposal immediately—and enthusiastically accepted it. At last there was a way to describe in mathematical terms such varied phenomena as coastlines, vascular systems, clouds, and star clusters. It seemed that by way of fractal geometry, any number of problems could be scientifically investigated for the first time.[7] "Without fractal geometry, there would be no language to describe the results, except to say that they look like a mess."[8]

Apparently, this new "language" was needed if we were to obtain a new view of many natural phenomena.[9] Whereas previously, *order* had been sought, now it was discovered that there existed "disorder on all fronts."[10] Indeed, the availability of fractal geometry appears to have had enormous effects on the way we look at nature. In what follows, I mean to show that a whole new concept of nature was shaped by Mandelbrot's theory.[11]

Places Often Imagined Before

We have been oversold on nature's mathematical regularity.[12]
—Stephen Jay Gould

Just what was the new concept of nature introduced by Mandelbrot, and how did it differ from earlier concepts? These questions are illuminated by an advertisement produced by IBM in 1987 (fig. 1). It presents a computer graphic resembling a mountain landscape, with ridge lines, snow-covered peaks, and a lake in the valley below. The caption boasts: "This place is really nowhere." The accompanying text explains that the company gives its outstanding researchers considerable freedom because "the freedom to explore ideas can lead to places never imagined before." In connection with the mountain landscape, to be sure, these statements appear to overlook what makes Mandelbrot's graphics special. For what is new and exciting about them is hardly that they picture places that are "really nowhere"—one could say the same thing about any given landscape by Claude Lorrain (1600–1682). For centuries, it was taken for granted that landscape pictures had no corollary in reality: They were inventions that might rely on nature studies in their details but in their overall composition were subject solely to the laws of *vraisemblance*.[13] The second, metaphorical claim that this vista was "never imagined before" seems forced as well: One ordinarily imagines an ideal mountain landscape in virtually this same way.

Fig. 1. "This place is really nowhere," 1987. Advertisement for IBM featuring a fractal landscape image created by IBM physicist Dr. Richard Voss. Reprint Courtesy of International Business Machines Corporation, © 2012 International Business Machines Corporation. MB-15

Instead, what is actually expressed in these statements is astonishment at the birth of a new kind of picture, one that had not existed before, that looked amazingly like nature but was entirely created by a computer. It had to seem like a minor miracle that with the proper programming, totally abstract fractal algorithms could create an image of mountains and valleys like those seen in the Alps or the Rockies.[14] "Constantly," Mandelbrot marveled in reference to another, equally transporting fractal landscape, "I lapse into wondering during which trip I actually saw the vista. . . ."[15] But what does this verisimilitude mean? Could it possibly indicate an inner relationship between fractal geometry and nature? This is the kind of attractive but at the same time highly problematical conjecture that gives fractal geometry its allure.

In fact, "nature" is perhaps the most important term in Mandelbrot's theory; for him, it is nature that allegedly obeys a fractal logic and produces fractals. Mandelbrot's credo was that "there is a fractal face to the geometry of nature." He has quite explicitly criticized existing theories of geometry that have "turned away from nature."[16] Just how a landscape like the one in the IBM advertisement was created is shifted somewhat into the background. Not every fractal algorithm ultimately produced mountains and valleys. A long process of experimentation with various functions, the tiniest variations on the so-called "Hausdorff Dimension" D and various graphic options, produced a huge number of pictures of which only a very few ultimately resembled landscapes (see Found Images III, cats 1–8).[17] For that reason, there were fundamental doubts about Mandelbrot's theory early on. Mathematicians argued that his graphics had little to do with nature—that the only thing one could see and study in them were the (admittedly highly complex and very beautiful) intrinsic geometric shapes of algorithms.[18] There was also harsh criticism from art historians: To assume an essential structural similarity based on visual resemblance was seen to be an argument by analogy of the "crudest sort."[19]

Nevertheless, the suspicion prevailed that the sciences were being offered enormous opportunities. And quite apart from the issue of whether nature in fact obeys a fractal logic, one thing can be stated with certainty: Fractal geometry has effected a paradigm shift in our concept of nature.[20] For example, in the IBM ad from 1987 it was already assumed that natural objects are irregular: "Now scientists and artists can create computer images—like this mountain—that have all the quirks and irregularities of natural objects." This was a rejection of the postulate that nature is essentially orderly and always produces predictable forms, which had stood unchallenged since the beginning of the modern era.[21] As the philosopher Alfred North Whitehead (1861–1947) wrote in 1926, without the widespread conviction that there is an order to nature, there could have been no science.[22] But with the advent of fractal geometry—together with that of contemporary chaos theory, which during the 1980s was becoming increasingly prevalent[23]—scientists began to see a *disorderly* nature everywhere they looked. Reality itself had become irregular, and with the awareness of fractal geometry our view of it had, as it were, lost its innocence: "Only recently," it was noted in 1997, "there was no word to describe fractals. Today we are beginning to see such features everywhere. Tomorrow, we may look at the entire universe through a fractal lens."[24]

Landscapes were now being seen through a fractal lens as well. Suddenly, it seemed that one had been searching for geomorphological regularities for far too long. A new view was required, one that took into account the irregular aspects of land forms as well. Geoscientists were now convinced that "certain phenomena

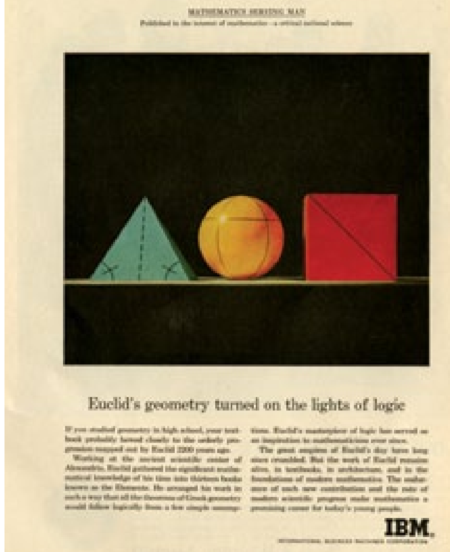

Fig. 2. "Euclid's geometry turned on the lights of logic," 1960. Advertisement for IBM. Reprint Courtesy of International Business Machines Corporation, © 2012 International Business Machines Corporation. MB-14

refuse to fit into a regular perspective."[25] Comparison with another IBM advertisement from a quarter-century earlier, in which the computer company seems to celebrate its mastery of the Euclidean paradigm, sheds light on the major changes in our thinking and observation that have taken place since then (fig. 2). The clear volumes seen in this 1960s ad—on which one could easily measure angles, place diagonals, and designate meridians—represented the basic building blocks of the scientific imagination. But the word "nature" was nowhere in evidence—it seemed that with logic alone, however brilliant, it was impossible to construct mountains.

Ill-figur'd Mountains

The nineteenth-century mathematicians may have been lacking in imagination, but nature was not.[26]
—Freeman Dyson

Mountains have always been difficult to fit into the order of nature. Using them as examples, it is possible to show how greatly

our views of nature have changed over time—and also how those views have been dictated by prevailing geometry.

Early on, it was noted that mountains appear to be irregular, even chaotic. After a journey through the Alps, the English theologian Thomas Burnet (1635–1715) could assert that "there is nothing in Nature more shapeless and ill-figur'd than an old Rock or a Mountain, and all that variety that is among them, is but the various modes of irregularity."[27] In his *Telluris theoria sacra*, first published in 1681 and repeatedly reprinted up into the nineteenth century, Burnet tried to dispense with the prevailing doctrine that the earth was regularly formed. Mountains served him as a prime example, for "they have neither form nor beauty, nor shape, nor order, no more than the Clouds in the Air." In Burnet, one of the fundamental problems with disorder becomes apparent: Disorder is difficult to describe. Irregularity is defined as the absence of order and pattern, but there were no positive terms with which Burnet might characterize it. This was still the case even in the eighteenth century: The terminology available to Geneva's natural philosopher and Alpine explorer Horace-Bénédict de Saussure (1740–1799), for example, was equally limited.[28]

Beginning in the nineteenth century, scholars were no longer willing to accept the irregularity of mountains. Geographers and geologists searched for regular structures behind their disorder, which they took to be only superficial. Such profound skepticism regarding the world of our direct perception was surely in part owing to Kant's distinction between the "thing in itself' and "things as they appear."[29] The German geographer Carl Ritter insistently warned against "taking the impression that strikes our senses for the object that produces it." One was by no means supposed to infer from such apparent confusion and lawlessness the actual absence of coherence and order. According to Ritter, what was needed was a "more profound consideration" of phenomena. Behind the planet's seemingly chaotic exterior there were doubtless higher symmetries and harmonies.[30]

The language available to scientists for the discovery of such symmetries and harmonies was geometry. Already, Ritter argued, the perfectly geometrical forms of cobwebs, plant cells, and crystals had been seen under the microscope. Geologists now went out in search of such regular patterns on a larger scale. Léonce Élie de Beaumont (1798–1874), one of the most influential figures in French science, postulated huge mountain systems that spread a pentagonal network—the so-called *réseau pentagonal*—across the entire globe (fig. 3). From the relative position of these mountain chains he hoped to be able to determine their age and reconstruct the conditions that had given rise to them.[31]

Fig. 3. Léonce Élie de Beaumont. "Le pentagone européen," 1852. Bibliothèque de l'École des mines de Paris, MINES ParisTech. MB-12

The belief that nature obeys geometrical principles runs through the entire nineteenth century. Perhaps it was most pointedly expressed by the French architect Eugène Viollet-le-Duc (1814–1879), who engaged in geology for many years. While studying Mont Blauen, he noted that it was highly fortunate that geometry was invented before the worlds were created, "for without it, building these worlds would have been impossible" (fig. 4).[32]

Needless to say, such views decisively influenced the contemporary view of nature. And even though today such theories as Élie de Beaumont's *réseau pentagonal*

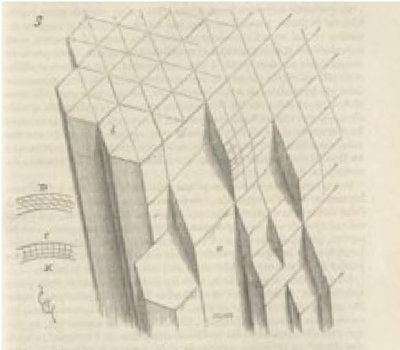

Fig. 4. Eugène Viollet-le-Duc. "Disposition des rhomboèdres granitiques." From Eugène Viollet-le-Duc, *Dictionnaire raisoné de l'architecture*, vol. 8, (Paris 1868), 481. MB-13

may strike us as absurd, they laid the groundwork for the tectonic theories of the twentieth century.[33] Plate tectonics, which finally came to be an acceptable geological theory in the 1960s, is still based on vast global structures that are only indirectly deduced from the planet's outward appearance. Although there was no longer any hope of finding perfect symmetries, to say nothing of pentagons, it was still possible to believe that one might find clearly defined forms that were compatible with Euclidean geometry.

Pattern Without Regularity

It is messy, and in the right way.[34]
—Michael Marder

One can now imagine what made fractal geometry seem like such an enormous provocation. In a sense, Mandelbrot was returning to the pre-Kantian theories of the seventeenth and eighteenth centuries. He ignored the whole distinction between "things as appearances" and "things in themselves," which had been a basic premise of modern scientific practice. Instead, like Burnet, he took the irregularities of natural phenomena at face value: Nature not only appears to be "irregular and fragmented," it *is* so. But unlike Burnet, he did not grow speechless in the face of disorder. On the contrary, with fractal geometry he developed a languagefor describing formerly indescribable phenomena. It is only his "morphology of the amorphous" that makes it possible for science to deal with disorder.[35]

To be sure, the irregularity of nature that Mandelbrot postulates can hardly be compared to that of the seventeenth and eighteenth centuries. It cannot be simply chance or "noise," otherwise it could not be captured in algorithms. Mandelbrot's achievement consists of having given a structure—a pattern—to fractal nature. This pattern is called "self-similarity." One recognizes fractals not in the forms themselves but in the invariance of their forms in scale: It does not matter how closely one zooms into the graphics, the same elements appear again and again.[36] Up close, it is claimed, coastlines would look precisely as irregular and random as from a great distance. Self-similarity seems like a revival of the notion of the *macrocosmos in microcosmo* so beloved by the ancients and by the hermeticists of early modern times, the idea that the whole cosmos is mirrored in the tiniest of objects and organisms.[37] But identifying it as the inherent pattern of natural irregularity—that would wait for Mandelbrot's insight.

Scientists have had to become accustomed to this unfamiliar "idea of pattern without regularity," which is also essential to the theory of deterministic chaos.[38] By now it appears that the "fractal lens" can hardly be set aside: Today, nature unquestionably appears to us as far "rougher" than it did half a century ago. It almost seems as if Mandelbrot had not only invented a new geometry—but a new nature as well.

1 I could not have written this text if Nina Samuel had not permitted me to read her dissertation on Mandelbrot before publication: many thanks to her. I also thank Katja Müller-Helle for her critical comments and helpful suggestions.
2 Benoît Mandelbrot, *The Fractal Geometry of Nature* (New York: Freeman, 1982), 262.
3 http://www.ibm.com/ibm100/us/en/icons/fractal/ (accessed March 28, 2012).
4 Benoît Mandelbrot, *Fractals: Form, Chance, and Dimension* (San Francisco: Freeman, 1977), 1.
5 Mathematicians here speak of functions that are constant but nowhere differentiable.
6 See Freeman Dyson, "Characterizing Irregularity: Review of Fractals. *Form, Chance and Dimension* by Benoît B. Mandelbrot," *Science* 200 (1978). Mathematicians frequently described functions like the Peano Curve as being "pathological"; see Mandelbrot, *The Fractal Geometry of Nature*, 59.
7 "It's only with Mandelbrot's insight . . . that physicists were able eventually to start to attack those problems." Michael F. Shlesinger, cited in Ivars Peterson, "Fractal Past, Fractal Future," *Science News*, no. 9 (1997): S13.
8 Michael Marder, "Roughing It," *Science* 277 (1997): 647.
9 Fractal geometry has often been described as a language; see, for example, Heinz-Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe, *Chaos and Fractals: New Frontiers of Science* (New York: Springer, 1989), 216.
10 Mark Buchanan, "The Irregularity of Reality," *Nature Physics* 7 (2011): 184.
11 For changes in the concept of nature throughout history, see Dieter Groh and Ruth Groh, *Weltbild und Naturaneignung. Zur Kulturgeschichte der Natur* (Frankfurt am Main: Suhrkamp, 1991); also, Götz Grossklaus and Ernst Oldemeyer, eds., *Natur als Gegenwelt. Beiträge zur Kulturgeschichte der Natur* (Karlsruhe: Loeper, 1983).
12 Stephen Jay Gould, *Questioning the Millennium: A Rationalist's Guide to a Precisely Arbitrary Countdown* (London: Vintage, 1998), 120.
13 See, for example, Oskar Bätschmann, *Entfernung der Natur. Landschaftsmalerei 1750–1920* (Cologne: DuMont, 1989). The Netherlandish tradition of landscape painting is an exception; see the exemplary discussion in Max Imdahl, "Jacob van Ruisdaels 'Die Mühle von Wijk,'" in *Max Imdahl: Gesammelte Schriften*, ed. Gundolf Winter (Frankfurt am Main: Suhrkamp, 1996).
14 See Nina Samuel, "'I look, look, look, and play with many pictures,' Zur Bilderfrage in Benoît Mandelbrot's Werk," in *Verwandte Bilder. Die Fragen der Bildwissenschaft*, ed. Ingeborg Reichle, Steffen Siegel, and Achim Spelten (Berlin: Kadmos, 2007); ibid., 271.
15 Mandelbrot, *The Fractal Geometry of Nature*, 271.
16 Ibid., 3, 1.
17 Among other ways, fractals are distinguished by the fact that they have a non-integral dimension D (in contrast to such traditional geometrical figures as line, surface, or solid, which always have integral dimensions). For his mountains, Mandelbrot varied the dimension between D=2.1 and D=2.5; see Mandelbrot, *The Fractal Geometry of Nature*, 256–71.
18 See James W. Cannon, "Review of *The Fractal Geometry of Nature* by Benoît B. Mandelbrot," *The American Mathematical Monthly* 91 (1984).
19 Horst Bredekamp, "Mimesis, grundlos," *Kunstforum International* 114 (1991): 278–88. **20** For the concept of the "paradigm shift," see Thomas S. Kuhn, *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 1962).
21 See Lorraine Daston and Michael Stolleis, eds., *Natural Law and Laws of Nature in Early Modern Europe: Jurisprudence, Theology, Moral, and Natural Philosophy* (Farnham: Ashgate, 2008) and Michael Hampe, *Eine kleine Geschichte des Naturgesetzbegriffs* (Frankfurt am Main: Suhrkamp, 2007).
22 Alfred North Whitehead, *Science and the Modern World* (Cambridge: Cambridge University Press, 1926), 4.
23 See James Gleick, *Chaos: Making a New Science* (New York: Viking, 1987).
24 Peterson, "Fractal Past, Fractal Future," S13.
25 W. E. H. Culling, "A New View of the Landscape," *Transactions of the Institute of British Geographers* 13 (1988): 346.
26 Dyson, "Characterizing Irregularity," 678.
27 Thomas Burnet, *The Sacred Theory of the Earth, Containing an Account of the Original of the Earth* (London, 1681), 98.
28 See, for example, Horace-Bénédict de Saussure, *Voyages dans les Alpes, précédés d'un essai sur l'histoire naturelle des environs de Genève* (Neuchâtel: Samuel Fauche, 1796),179. For Saussure, see also Claude Reichler, *La découverte des Alpes et la question du paysage* (Geneva: Georg Editeur, 2002).
29 Immanuel Kant, *Kritik der reinen Vernunft* (Riga: Hartknoch, 1787), xxv. For the influence of Kant's distinction on scientific observation, see also Jonathan Crary, *Techniques of the Observer: On Vision and Modernity in the Nineteenth Century* (Cambridge, MA: MIT Press, 1990), 69.
30 Carl Ritter, "Ueber räumliche Anordnungen auf der Aussenseite des Erdballs und ihre Functionen im Entwicklungsgange der Geschichten," in *Einleitung zur allgemeinen vergleichenden Geographie*, ed. Carl Ritter (Berlin: Georg Reimer, 1852), 209–10.
31 Léonce Élie de Beaumont, *Notice sur les systèmes des montagnes*, 3 vols. (Paris: P. Bertrand, 1852). See also Barbara A. Kennedy, *Inventing the Earth: Ideas on Landscape Development since 1740* (Malden, MA: Blackwell, 2006), 10; and Jaques Touret, "Élie de Beaumont (1798–1874), des systèmes de montagnes au réseau pentagonal," in *L'essor de la géologie française*, ed. Jean Gaudant (Paris: Presses des Mines, 2009).
32 Viollet-le-Duc, quoted from Pierra A. Frey and Lise Grenier, eds., *Viollet-le-Duc et la Montagne* (Grenoble: Glénat, 1993), 29. For his geological studies, see Vio Eugène Emmanuel Viollet-le-Duc, *Le Massif du Mont-Blanc.* (Paris: Baudry, 1876).
33 See Mott T. Greene, *Geology in the Nineteenth Century: Changing Views of a Changing World* (Ithaca, NY: Cornell University Press, 1982), 77.
34 Marder, "Roughing It," 647.
35 Mandelbrot, *The Fractal Geometry of Nature*, 1.
36 A demonstration video of a zoom into a Mandelbrot set can be found on the IBM Web site: http://www.ibm.com/ibm100/us/en/icons/fractal/.
37 See Robert Fludd, *The Origin and Structure of the Cosmos*, trans. Patricia Tahil (Edinburgh: Magnum Opus Hermetic Sourceworks, 1982). This was also a fundamental concept behind the creation of the modern-art and curio cabinet: See Andreas Grote, ed., *Macrocosmos in Microcosmo: Die Welt in der Stube. Zur Geschichte des Sammelns 1450 bis 1800* (Oplanden: Leske + Budrich, 1994).
38 W. E. H. Culling, "A New View of the Landscape," *Transactions of the Institute of British Geographers* 13 (1988), 357.